

Predicting Stellar Angular Sizes

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Abstract. Reliable prediction of stellar diameters, particularly angular diameters, is a useful and necessary tool for the increasing number of milliarcsecond resolution studies being carried out in the astronomical community. A new and accurate technique of predicting angular sizes is presented for giant and supergiant stars, and for more evolved sources such as carbon stars and Mira variables. This technique uses observed K and either V or B broad-band photometry to predict $V = 0$ or $B = 0$ zero magnitude angular sizes, which are then readily scaled to the apparent angular sizes with the V or B photometry. The spread in the relationship is 11-12% for giant and supergiant stars, and for evolved sources, results are at the 20-26% level. Compared to other simple predictions of angular size, such as linear radius-distance methods or black-body estimates, zero magnitude angular size predictions can provide apparent angular sizes with errors that are 2 to 5 times smaller.

1. Introduction

Prediction of stellar angular sizes is a tool that has come to be used with greater frequency with the advent of high resolution astronomical instrumentation. Structure at the tens to hundreds of milliarcsecond (mas) level is now being routinely observed with the Hubble Space Telescope, speckle interferometry, and adaptive optic systems. Single milliarcsecond observations, selectively available for many years with the technique of lunar occultations, are now becoming less specialized as prototype interferometers in the optical and near infrared evolve towards facility class instruments. For all of these telescopes and techniques, it is often desirable to predict angular size of stars, to select either appropriate targets or calibration sources.

Detailed photometric and spectrophotometric predictive methods provide results with high accuracy (1-2% diameters; cf. Blackwell & Lynas-Gray 1998, Cohen et al. 1996). However, diameters from these methods require large amounts of data that is often difficult to obtain, and as such, are available for a limited number of objects. For the general sample of stars, only limited information is available, and spectral typing, photometry, and parallaxes are all less available and less accurate as one examines stars at greater distances. Deriving expected angular sizes is a greater challenge in this case. Fortunately the general availability of B or V band data, and forthcoming release of the data from the 2MASS and DENIS surveys, which have limiting magnitudes of

$K > 14.3$ and 13.5 , respectively (Beichman et al. 1998, Epchtein 1997), will provide at least broad-band photometry on these more distant sources. Given these databases, of general utility is a method based strictly upon this widely available data. In this paper a method based solely upon K and either B or V broad-band photometry will be presented, and it will be shown that angular sizes for a wide variety of sources can be robustly predicted with merely two-color information. A similar relationship is discussed by Mozurkewich et al. (1991), who present a 'distance normalized uniform disk angular diameter' as a function of $R - I$ color, but with a limited number ($N = 12$) of objects to calibrate the relationship. Related to these methods is the study of stellar surface brightness as a function of $V - K$ color published by Di Benedetto (1993), which built on the previous work by Barnes & Evans (Barnes & Evans 1976, Barnes, Evans & Moffett 1976, Barnes, Evans & Parsons 1978). A more detailed exposition of this technique is presented in van Belle (1999) and includes discussions covering main sequence stars and the minimal effect of interstellar extinction.

2. Zero Magnitude Angular Size versus $V - K$, $B - K$ Colors

The large body of angular sizes now available allows for direct predictions of expected angular sizes, bypassing many astrophysical considerations, such as atmospheric structure, distance, spectral type, reddening, and linear size. To compare angular sizes of stars at different distances, one approach is to scale the sizes relative to a zero magnitude of $V = 0$:

$$\theta_{V=0} = \theta \times 10^{V/5}. \quad (1)$$

The angular size thus becomes a measure of apparent surface brightness (a more detailed discussion of related quantities may be found in Di Benedetto 1993.) Conversion between a $V = 0$ zero magnitude angular size, $\theta_{V=0}$, and actual angular size, θ , is trivial with a known V magnitude and the equation above. The same approach has been employed for $K = 0$ (see Dyck et al. 1996a) and will also be applied in this paper to $B = 0$. Given the general prevalence of V band and the inclusion of B band data in the 2MASS catalog, the apparent angular size approach will be developed here for $V - K$ and $B - K$ colors.

2.1. Evolved Sources: Giant and Supergiant Stars

163 normal giant and supergiant stars found in the interferometry and lunar occultation papers were also found to have available V photometry. By examining their near-infrared angular sizes, we can establish a relationship between $V = 0$ zero magnitude angular size and $V - K$ color:

$$\theta_{V=0} = 10^{0.669 \pm 0.052 + 0.223 \pm 0.010 \times (V-K)}. \quad (2)$$

The errors on the 2 parameters in the equation above are 1σ errors determined from a χ^2 minimization; given 2 degrees of freedom in the equation, $\Delta\chi^2 = 2.30$ about the χ^2 minimum for this case (Press et al. 1992). Similar error calculations will be given for all other relationships reported in this manuscript. Examining the distribution of the differences between the fit and the measured

values, $\Delta\theta_{V=0}$, we find an approximately Gaussian distribution with the rms value of the 163 differences yielding a fractional error of $(\Delta\theta_{V=0}/\theta_{V=0})_{rms} = 11.7\%$.

Similarly, for $B - K$ color, 136 giant and supergiant stars had available photometry, resulting in the following fit:

$$\theta_{B=0} = 10^{0.648 \pm 0.072 + 0.220 \pm 0.012 \times (B-K)}, \quad (3)$$

with an rms error of 10.8%.

The relationship appears valid over a $V - K$ range of 2.0 to 8.0. Blueward of $V - K = 2.0$, the subsample is too small ($N = 3$) to confidently indicate whether or not the fit is valid, in spite of the goodness of fit for the whole subsample. The same is true redward of $V - K = 8.0$. Also, for stars redward of approximately $V - K = 8$, care must be taken to exclude variable stars (both semiregular and Miras). The data points and the fit noted above may be seen in Figure 1; $\theta_{V=0}$ and standard deviation by $V - K$ bin is given in Table 1. The Miras are plotted separately in Figure 2 and will be discussed below.

For $B - K$ between 3.0 and 7.5, the relationship exhibits a similar if not slightly superior validity. As with the $V - K$ color, the relationship appears to be valid down blueward of the short edge of that range, down to $B - K = -1$, but the data are sparse. Redward of $B - K = 7.5$, the relationship also exhibits potential confusion with the Mira variable stars, although there appears to be less degeneracy, but this is possibly due to a lesser availability of B band data on these very red sources. The data points and the fit noted above may be seen in Figure 3; $\theta_{B=0}$ and standard deviation by $B - K$ bin is given in Table 2.

The potential misclassification of more evolved sources such as carbon stars and variables (Miras or otherwise) as normal giant and supergiant stars is a significant secondary consideration. For the dimmer sources for which little data is available, non-classification is perhaps the more appropriate term. What is reassuring with regards to the issue of classification errors is that the robust relationships between $(\theta_{V=0}, V - K)$ and $(\theta_{B=0}, B - K)$ are valid for stars of luminosity class I, II, and III, and that the more evolved stars occupy a redder range of $B - K$ and $V - K$ colors (cf. §2.2.). Since the $\theta_{V=0}$ and $\theta_{B=0}$ relationships are insensitive to errors in luminosity class, this method is more robust than the linear radius-distance method, particularly for those stars in the $2.0 < V - K < 6.0$ and $3.0 < B - K < 7.5$ ranges, where few if any stars of significant variability exist. This relationship is also considerably easier to employ than the method of blackbody fits.

2.2. Evolved Sources: Variable Stars

By examining the $2.2 \mu\text{m}$ angular sizes for the 87 observations of 65 semiregular variables, Mira variables and carbon stars (broadly classified here as 'variable stars') found in the literature, we can establish a relationship between $V = 0$ zero magnitude angular size and $V - K$ color:

$$\theta_{V=0} = 10^{0.789 \pm 0.119 + 0.218 \pm 0.014 \times (V-K)}. \quad (4)$$

The rms error associated with this fit is 26%. The data points and the fit noted above may be seen in Figure 3. Similarly, for $B - K$ color, 19 evolved sources had

Table 1. Zero Magnitude Angular Size for Giants and Supergiants as a Function of $V - K$ Color.

Normal Giants and Supergiants					Variables				
$V - K$ Center	N	$\bar{\theta}_{V=0}$	Std. Dev.	Fit	N	$\bar{\theta}_{V=0}$	Std. Dev.	Fit	Ratio
-0.5	1	3.4		3.7	0				
0.0	0			4.8	0				
0.5	0			6.2	0				
1.0	1	9.1		8.0	0				
1.5	2	11.6	1.8	10.3	0				
2.0	9	13.9	1.7	13.3	0				
2.5	17	16.7	3.1	17.2	0				
3.0	12	20.5	3.1	22.3	0				
3.5	20	27.2	4.4	28.7	0				
4.0	21	37.8	4.4	37.1	0				
4.5	15	47.0	5.7	47.9	0				
5.0	18	58.2	5.9	61.9	0				
5.5	15	80.3	13.9	79.9	4	105	13	103	1.31 ± 0.28
6.0	7	102.7	13.3	103.1	7	140	25	132	1.37 ± 0.30
6.5	5	122.9	18.3	133.1	9	181	57	170	1.47 ± 0.51
7.0	9	159.6	23.5	171.9	8	233	60	220	1.46 ± 0.43
7.5	6	197.0	21.0	221.9	14	270	62	283	1.37 ± 0.35
8.0	0			286.6	9	461	184	365	
8.5	1	355.4		370.0	4	605	217	470	1.70 ± 0.61
9.0	1	431.0		477.7	7	631	245	605	1.46 ± 0.57
9.5	0				3	841	259	780	
10.0	0				2	1286	511	1005	
10.5	0				4	1456	604	1295	
11.0	0				6	1795	465	1669	
11.5	0				2	2146	498	2150	
12.0	0				0				
12.5	0				2	3033	965	3569	
13.0	0				0				
13.5	0				0				
14.0	0				1	8323		7635	

Notes: The number of stars N , average size $\theta_{V=0}$, and standard deviation for each bin is given for both normal giant and supergiant stars, and for variables, inclusive of Miras, semi-regulars, and carbon stars. The fits given are those discussed in §2.; the ratios given are the average $\theta_{V=0}$ size ratios for those $V - K$ bins where values exist for both giant/supergiant stars and variables. In general, the variable stars have a $\theta_{V=0}$ size that is 1.44 ± 0.15 larger than their 'normal' star counter parts for a given $V - K$ color.

Table 2. Zero Magnitude Angular Size for Giants and Supergiants as a Function of $B - K$ Color.

Normal Giants and Supergiants					Variables				
$B - K$	N	$\bar{\theta}_{B=0}$	Std.		N	$\bar{\theta}_{B=0}$	Std.		Ratio
Bin Center			Dev.	Fit			Dev.	Fit	
-0.5	1	3.2		3.5	0				
0.0	0			4.5	0				
0.5	0			5.8	0				
1.0	0			7.5	0				
1.5	1	10.9		9.7	0				
2.0	1	13.6		12.5	0				
2.5	1	18.7		16.1	0				
3.0	10	21.4	2.7	20.7	0				
3.5	13	26.2	4.2	26.7	0				
4.0	11	34.5	3.6	34.4	0				
4.5	6	47.2	8.2	44.3	0				
5.0	15	51.9	5.3	57.1	0				
5.5	14	74.9	11.5	73.6	0				
6.0	18	89.5	12.0	94.8	0				
6.5	12	114.4	19.3	122.1	0				
7.0	13	151.5	15.9	157.4	0				
7.5	6	196.1	21.3	202.8	0				
8.0	4	248.4	23.2	261.2	6	304	75	352	1.23 ± 0.32
8.5	7	315.6	21.8	336.6	3	451	84	447	1.43 ± 0.28
9.0	2	344.2	5.5	433.7	1	520		569	
9.5	0				5	669	164	723	
10.0	0				3	1057	273	919	
10.5	0				1	1270		1169	
11.0	0				0				
11.5	0				2	2501	561	1889	
12.0	0				2	2802	316	2402	
12.5	0				0				
13.0	0				1	3302		3883	
13.5	0				0				
14.0	0				1	5797		6276	
14.5	0				1	9077		7979	
15.0	0				2	12161	1755	10144	

Notes: The number of stars N , average size $\theta_{B=0}$, and standard deviation for each bin is given for both normal giant and supergiant stars, and for variables, inclusive of Miras, semi-regulars, and carbon stars. The fits given are those discussed in §2.; the ratios given are the average $\theta_{B=0}$ size ratios for those $B - K$ bins where values exist for both giant/supergiant stars and variables. In general, the variable stars have a $\theta_{B=0}$ size that is 1.34 ± 0.21 larger than their 'normal' star counter parts for a given $B - K$ color.

available photometry for 29 angular size observations, resulting in the following fit:

$$\theta_{B=0} = 10^{0.840 \pm 0.096 + 0.211 \pm 0.008 \times (B-K)}, \quad (5)$$

with a rms error of 20%.

For the variable stars, the relationship appears valid over $V-K$, $B-K$ ranges of 5.5 to 13.0 and 9.0 to 16.0, respectively. Redward of $V-K=13$, the sample is too small ($N=3$) to confidently indicate whether or not the fit is valid, in spite of the goodness of fit for the general sample. It is interesting to note that the slope of the fits for the variable stars and for the giant/supergiant stars is statistically identical for both $V-K$ and $B-K$ colors; only the intercepts are different. This corresponds to a $\theta_{V=0}$ size factor of 1.40 ± 0.15 between the smaller normal and the larger variable stars for a given $V-K$ color, and a corresponding $\theta_{B=0}$ size factor of 1.34 ± 0.21 .

2.3. Analysis of Errors

As was given in §2.1., the rms fractional error between the measured and predicted values for $\theta_{V=0}$ versus $V-K$ for giants and supergiants is $(\Delta\theta_{V=0}/\theta_{V=0})_{rms} = 11.7\%$. There are three components of this error: (1) Angular size errors, (2) Errors in $V-K$, and (3) Deviations in the relationship due to unparameterized phenomena, which shall be broadly labeled 'natural dispersion' in the relationship and will be discussed in more detail below. For the first component, the rms fractional error of the 163 measured θ values found in the literature is $(\Delta\theta/\theta)_{rms} = 6.9\%$. For the photometry, given the heterogeneous sources, we estimate that the V and K photometry will have errors between 0.1 and 0.2 magnitudes (resulting in $V-K$ color errors of 0.14 to 0.28 magnitudes), which would result in an size error of 3.1-6.3%. Finally, subtracting these two sources of measurement error in quadrature from the measured dispersion, a natural dispersion in the relationship between 7.0 and 8.9% remains. A similar analysis for the giant/supergiant $\theta_{B=0}$ versus $B-K$ results in $(\Delta\theta/\theta)_{rms} = 7.0\%$ for the 136 observations, indicating of 5.2-7.6% of natural dispersion. For both of these relationships, the natural dispersion is a factor as significant as the errors in angular size, and potentially the dominant factor.

For the variable stars, the difficulties in obtaining contemporaneous photometry result in larger measurement error, despite steps taken to ensure epoch-dependent observations. As such, the errors are expected to be between 0.2 and 0.4 magnitudes for the individual V and K measurements. The resulting characterization of natural dispersion of 20-23% for the $V-K$ relationship, and 12-16% for $B-K$, dominating the angular size dispersion of $(\Delta\theta/\theta)_{rms} = 10\%$ for both colors.

The specific nature of the natural dispersion term in the rms error is potentially due to stellar surface properties that affect current one-dimensional angular size determination techniques. The limited observations of individual objects with two-dimensional and more complete spatial frequency coverage have indicated asymmetries in stellar atmospheres that could potentially affect size determinations from both interferometric and lunar occultations. Early measurements of this nature were detection of asymmetries in the envelope of α Cen with speckle interferometry (Karovska et al. 1991). Direct imaging of the surface of α Ori has provided evidence of a large hot spot on that supergiant's surface

(Gilliland & Dupree 1996). More recently, similar evidence for aspheric shapes of other Miras has been obtained, also with HST (Lattanzi et al. 1997), and evidence for more complicated morphologies in the structure of the M5 supergiant VY CMa in the near-IR has been obtained using nonredundant aperture masking on Keck 1 (Monnier et al. 1999; Tuthill et al. 1999). Various atmospheric phenomena, such as nonradial pulsations, spots on the stellar surface, and rotational distortion of the stellar envelope, potentially explain these observations. The progressive increase along stellar evolutionary states in observed natural dispersion from slight levels with the giant/supergiant stars to dominant levels with the most evolved sources is consistent with the onset of these phenomena more significantly associated with extended atmospheres.

Table 3. Comparison of the Various Angular Size Prediction Methods..

	Method	Error	Notes
Giant, Supergiant Stars	Linear Radius by Spectral Type	22%	Giants only
	Linear Radius by $V - K$ Color	22%	Giants only
	Angular Size by BBR Fit	18%	
	$V = 0$ Angular Size by $V - K$ Color	11.7%	
	$B = 0$ Angular Size by $B - K$ Color	10.8%	
Variable Stars	$V = 0$ Angular Size by $V - K$ Color	26%	
	$B = 0$ Angular Size by $B - K$ Color	20%	

Notes: Errors given above are percentage errors relative to the value predicted by each method.

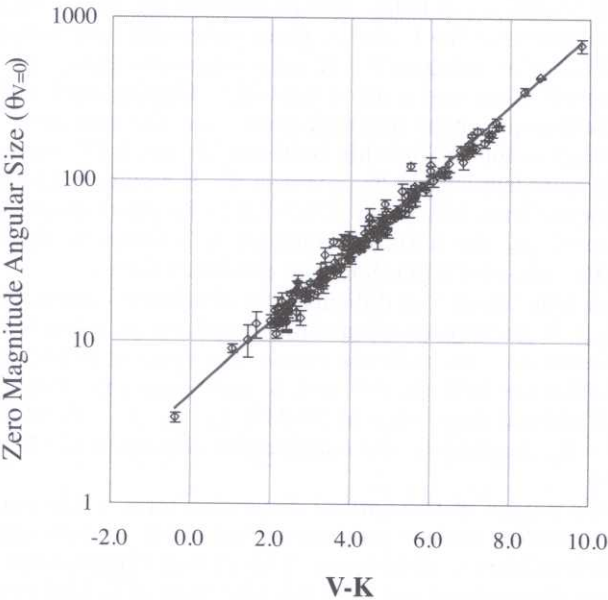


Figure 1. The $\theta_{V=0}$ zero magnitude angular size versus $V - K$ color for luminosity class I, II, and III giant stars.

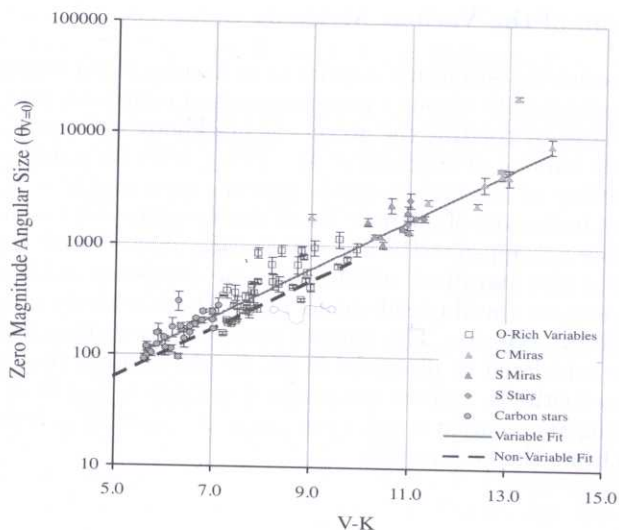


Figure 2. The $\theta_{V=0}$ zero magnitude angular size versus $V - K$ color for evolved stars, including carbon stars, S stars, all types of Mira variables, and non-Mira variables. The solid upper line is the fit line for these objects, and the dashed lower line is the fit line for the giants and supergiants from Figure 1.

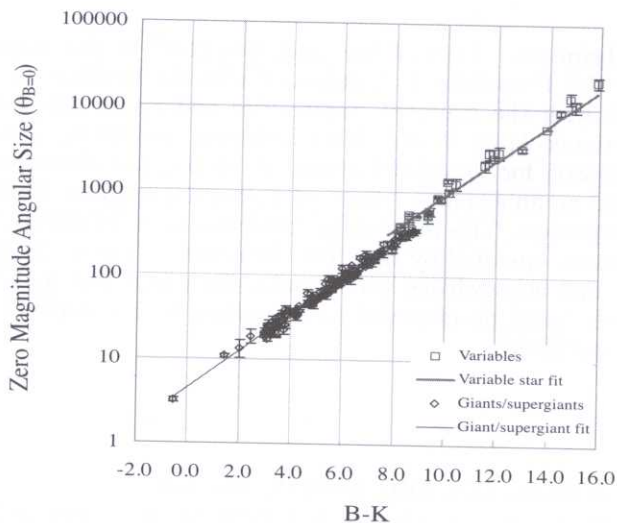


Figure 3. The $\theta_{B=0}$ zero magnitude angular size versus $B - K$ color for giant/supergiant stars and evolved stars, which includes Mira variables, S stars, carbon stars, and non-Mira variables. The upper line is the fit line for the evolved stars, the lower line is the fit line for the giants and supergiants.

3. Comparison of the Various Methods

Previous approaches for estimating angular sizes have included estimates of stellar linear size coupled with distance measurements or estimates, and the extraction of angular sizes by treating the objects as blackbody radiators. The release of the Hipparcos catalog (Perryman et al. 1997), with its parallax data, has increased the utility of the first method. Spectral type and $V - K$ color have been explored as indicators of intrinsic linear size for giant stars (van Belle et al. 1999). There does not appear to be a $V - K$ -linear radius relationship presented for these stars in the literature, which would be consistent with both photometric bands being on the Rayleigh-Jeans tail of the blackbody curve for these hotter ($T > 6000K$) objects. The relative errors for predicting stellar angular diameters were calculated as discussed in §2. for the stars in the sample using these alternative methods, and are summarized in Table 3. For all of the stars in question, deriving an apparent angular size from a $\theta_{V=0}$ or $\theta_{B=0}$ zero magnitude angular size delivers the best results.

4. Conclusion

The new approach of establishing the $\theta_{V=0}$ and $\theta_{B=0}$ zero magnitude angular sizes appears to be an unrecognized yet powerful tool for predicting the apparent angular sizes of stars of all classes. The very modest data requirements of this method make it an ideal tool for quantification of this fundamental stellar parameter.

Acknowledgments. Part of the work described in this paper was performed at the Jet Propulsion Laboratory, California Institute of Technology under contract with the National Aeronautics and Space Administration. I would like to thank Andy Boden, Mark Colavita, Mel Dyck, Steve Ridgway, and Bob Thompson for thoughtful comments during the development of this manuscript, and an anonymous referee who provided valuable feedback during the publication process. This research has made use of the SIMBAD, VizieR, and AFOEV databases, operated by the CDS, Strasbourg, France. In this research, we have used, and acknowledge with thanks, data from the AAVSO International Database, based on observations submitted to the AAVSO by variable star observers worldwide.

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